One warning, however, for those at colleges with a somewhat less select student body than Amherst's. This is a valuable addition to the available texts in the philosophy of mathematics and, at the same time, a great disappointment.

The disappointing part. When I was asked to review the text, I thought, "Wonderful! In the early part of the 20th century, the serious concerns about the foundations of mathematics pushed aside all philosophical questions concerning mathematics except foundational ones, and only in the last 30 years or so has there been a rebirth of the subject. We really need a text which takes the philosophy of mathematics to the modern era and shows it's still a growing field. I'm looking forward to reading a book which moves on to the exciting new ideas philosophers have been discussing in recent years: Hartry Field's attempts to develop physics in the absence of mathematics to do away with the indispensibility argument for the existence of mathematical objects; Michael Resnik's, Stewart Shapiro's and Geoffrey Hellman's attempts to give a structuralist account of mathematics as a science of patterns; Paul Ernest's and Philip Davis and Reuben Hersh's work on social constructivism applied to mathematics; and the work of John Burgess, Kenneth Manders, Penelope Maddy, Mark Steiner, Bob Hale, Mark Balaguer, and many others."

Alas, this is not the text I've been waiting for. With the exception of a few remarks here and there, this text could have been written 50 years ago. It goes over the same old ground: logicism, intuitionism, and formalism (called, in the text, and much more appropriately, "finitism"), the three alternatives to realism introduced around the turn of the 20th century. From this book's point of view, the subject has been essentially moribund for the last 50 years.

Yet a valuable addition. On the other hand, no one did write a text such as this 50 years ago. There are a few books one could use as a text for a course in the foundations of mathematics: there's a nice book by Stephan Körner (The Philosophy of Mathematics: an Introductory Essay, first published in 1960 by Hutchinson University Library and now available from Dover Publications) which does discuss in fairly good detail the three foundationalist views. However, George and Velleman's book is a much more detailed exposition of these viewpoints than any I've seen, and the first which really is an appropriate text for use with undergraduates or beginning graduate students. The authors explain all three viewpoints very thoroughly, bringing out all kinds of subtleties which beginning students of the subject are likely to miss or become confused by. For example, in the discussion of intuitionism, p. 106, after they have discussed some of the kinds of reasoning intuitionists reject, and why, there's the following passage:

"First, note that the difference between the classical mathematician and the intuitionist is not that the one makes infinitary claims while the other refrains from doing so. They both find infinitary assertions intelligible; they differ, however, in the assertibility conditions that they associate with such claims. Secondly, it would be a mistake to think intuitionism differs from realism in judging statements that have not yet been proved to be unintelligible. One need not have proved a statement in order to understand it; for the intuitionist, it suffices to know what would count as a proof of it. For example, (for all x)(there exists y)(x + y = 0 implies 2 + x) [which, to a realist means there are an infinite number of twin primes, a conjecture which hasn't yet been proven] although thus far undecided, is intelligible to the intuitionist because" [as they did on the page previous to the one quoted] one can specify precisely what its proof conditions are."

One advantage in using this book as a text for a course in the foundations of mathematics is that it goes beyond presenting three foundationalist approaches to give a quite thorough discussion of the incompleteness theorems, both Gödel's original work and later advances, and their effect on the three foundationalist programs. I'm less delighted with their exposition of the incompleteness theorems than with other parts of the book — I've seen a few others which seem to me a bit easier for students to follow — but they do an unusually good job of presenting the rationale behind what is being done before going into the gory details.

An additional nice feature are the exercises at the end of the chapter, which should be approachable by students who can read the text, and which extend some of the discussion in the text.

One warning, however, for those at colleges with a somewhat less select student body than Amherst's. This is a...
A text for students at a college such as Amherst (where the authors teach), or for graduate students. While formally a student doesn't have to have studied much mathematics, nor much philosophy, beyond a fairly thorough grounding in logic, there are many parts (particularly in the discussion of intuitionistic mathematics and of Gödel's incompleteness theorems) of the book where the average undergraduate will find the details very hard to follow. Indeed, it's necessary to read this book as one would any mathematics book, with pencil and paper in hand to fill in the details. It requires a considerable level of mathematical sophistication. On the other hand, perhaps it's not reasonable to try to teach a course on the philosophy of mathematics to students who have no feeling for the subject whose philosophy is being studied.

Thus, while this book is disappointing for what it doesn't do, it is a very valuable addition to the tiny pile of texts which can be used in a course in the philosophy of mathematics.

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Preface.

1. Introduction.
2. Logicism.
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5. Intuitionistic Mathematics.
6. Finitism.
7. The Incompleteness Theorems.
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The philosophy of mathematics is the branch of philosophy that studies the philosophical assumptions, foundations, and implications of mathematics. The aim of the philosophy of mathematics is to provide an account of the nature and methodology of mathematics and to understand the place of mathematics in people's lives. ✪ Intro to the Philosophy of Mathematics (Ray Monk). ✪ What are Numbers? Philosophy of Mathematics. ✪ Philosophy of Mathematics: Platonism. The philosophy of mathematics is the branch of philosophy which deals with the philosophical foundations of mathematics. Some of the major viewpoints include: Platonism: the position that mathematical objects (numbers, sets, fields, etc.) have a real existence independent from the physical world, that they are real existing immaterial objects. Formalism: the position that mathematics is nothing more than the manipulation of symbols according to formal rules; those symbols are not ascribed any ultimate